ACT Government

Predicting phosphorus concentration and trophic state of Lake Tuggeranong

Technical report supplement to the Lake Tuggeranong Catchment Model

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Contents

1. Introduction

Vollenweider's (1975, 1976) mass-balance equations, overall, model how the total phosphorous (TP) concentration of a lake ($\mathrm{TP_{lake}}$) is related to the TP concentration of inflows ($\mathrm{TP_{in}}$) and the hydraulic residence/retention time (τ_w). This residence time can be thought of as the average time an incoming water molecule can expect to remain in the lake before leaving the lake through outflow and is calculated by simply dividing the volume of the lake by the outflow rate.

Vollenweider (1976 – Equation 2.13) described this relationship mathematically with the equation below (as re-stated in Brett & Benjamin 2008 – Equation 6):

$$\mathrm{TP_{lake}} = \frac{\mathrm{TP_{in}}}{1+\sigma\tau_w} \qquad (1)$$

Where σ is the first-order rate coefficient of TP loss within the lake due to sedimentation. As this rate is not easily quantified, several different approaches have been proposed to approximate it. Brett & Benjamin (2008) summarised and compared the relative performance of these different approaches using known values from 305 lakes from Europe and North America. They found that all approaches provided a useful fit with real-world values, but that the best-fitting approach was to assume that σ is a function of $k{\tau_w}^x$, where k and x are constants, fitted statistically. This approach was previously used by Larsen & Mercier (1976), who reported a value of x near -0.5.

With this approximation, equation 1 becomes:

$$\mathrm{TP_{lake}} = \frac{\mathrm{TP_{in}}}{1+(k{\tau_w}^x)\tau_w} \qquad (2)$$

Brett & Benjamin (2008) found that values of around k=1.12 and x=-0.53 yielded a correlation between predicted and observed TP_{lake} values with r^2>0.8 for the dataset investigated, yielding the following relationship:

$$\mathrm{TP_{lake}} = \frac{\mathrm{TP_{in}}}{1+\left(1.12\times\tau_w^{-0.53}\right)*\tau_w} \qquad (3)$$

If we then approximate k=1 and x=-0.5, equation 3 simplifies to:

$$\mathrm{TP_{lake}} = \frac{\mathrm{TP_{in}}}{1+\sqrt{\tau_w}} \qquad (4)$$

Vollenweider (1976) previously suggested this approximation of the general equation fits with empirical studies, and this relationship was used by Beer *et al.* 1982 in their early work on Lake Tuggeranong.

2. In-Lake Model

2.1 Parameterisation of the modified Vollenweider model

The adjusted Vollenweider model (equation 2) was parameterised using estimates of inflow TP concentrations and inflow and outflow volumes from the Lake Tuggeranong catchment model (Dey et al. 2024), and observed Lake Tuggeranong TP concentrations from the ALS Lakes and Rivers Water Quality Monitoring Program, but with the following caveats:

- The model assumes a steady state of inflows and influent concentrations, which is a limitation to real-world applications, so it is unclear how reliable we should expect its predictions over shorter time periods to be. That being said, past modelling of TP loads has suggested that 3 hydraulic residence times is sufficient for 95% convergence with steady-state predictions (Sonzogni *et al.* 1976), which is <1 year for most years in Lake Tuggeranong. For this reason, our modelling approach will work with annual time steps, with all model inputs and outputs to be interpreted as annual averages.
- In past applications of this model-fitting approach, data across many different lakes were used, with each lake representing a separated datapoint (e.g. Brett & Benjamin 2008, Larsen & Mercier 1976). Here, the datapoints are all from Lake Tuggeranong, just from different years, and are thus pseudo-replicated. However, to provide a Lake Tuggeranong -specific model that will not be applied elsewhere this approach is probably reasonable. It would be informative to repeat this modelling approach in other ACT lakes and compare fitted values later, to investigate the appropriateness of an overall ACT-wide model using data from all different lakes.
- In other applications of Vollenweider model the same value for water inflow and outflow are used (i.e. assumption of no other source of water loss). Since the catchment model have taken into account of the lake water balance.

To parametrise the modified Vollenweider model, we can use annual total TP loads and water inflows and outflows from the Tuggeranong Source Catchment Model. Specifically:

- l_{in}: the annual TP load flowing into Lake Tuggeranong (converted to mg),
- v_{in}: the annual inflow volume of water into Lake Tuggeranong (converted to L)
- v_{out}: the annual outflow volume of water from Lake Tuggeranong (converted to L)

We can then combine these parameters, along with the approximate total volume of Lake Tuggeranong (V= 1,780,000,000L), to calculate the TP concentration of inflows ($\mathrm{TP}_{\mathrm{in}}$, measured in mg/L) and the hydraulic residence time (τ_w, measured in years) as follows:

$$\mathrm{TP}_{\mathrm{in}} = \frac{l_{\mathrm{in}}}{v_{\mathrm{in}}} \qquad (5)$$

$$\tau_w = \frac{V}{v_{\mathrm{out}}} \qquad (6)$$

The predicted annual average $\mathrm{TP}_{\mathrm{in}}$ and τ_w values from 1990-2022 are summarised in Table 1 and shown in Figure 1.

Table 1. Summary of Lake Tuggeranong annual values of inflow total phosphorus concentration TP_{in} and hydraulic residence time τ_w from 1990-2022.

	TP_{in} (mg/L)	τ_w (y)
Minimum	0.152	0.055
Median	0.163	0.251
Maximum	0.174	0.727

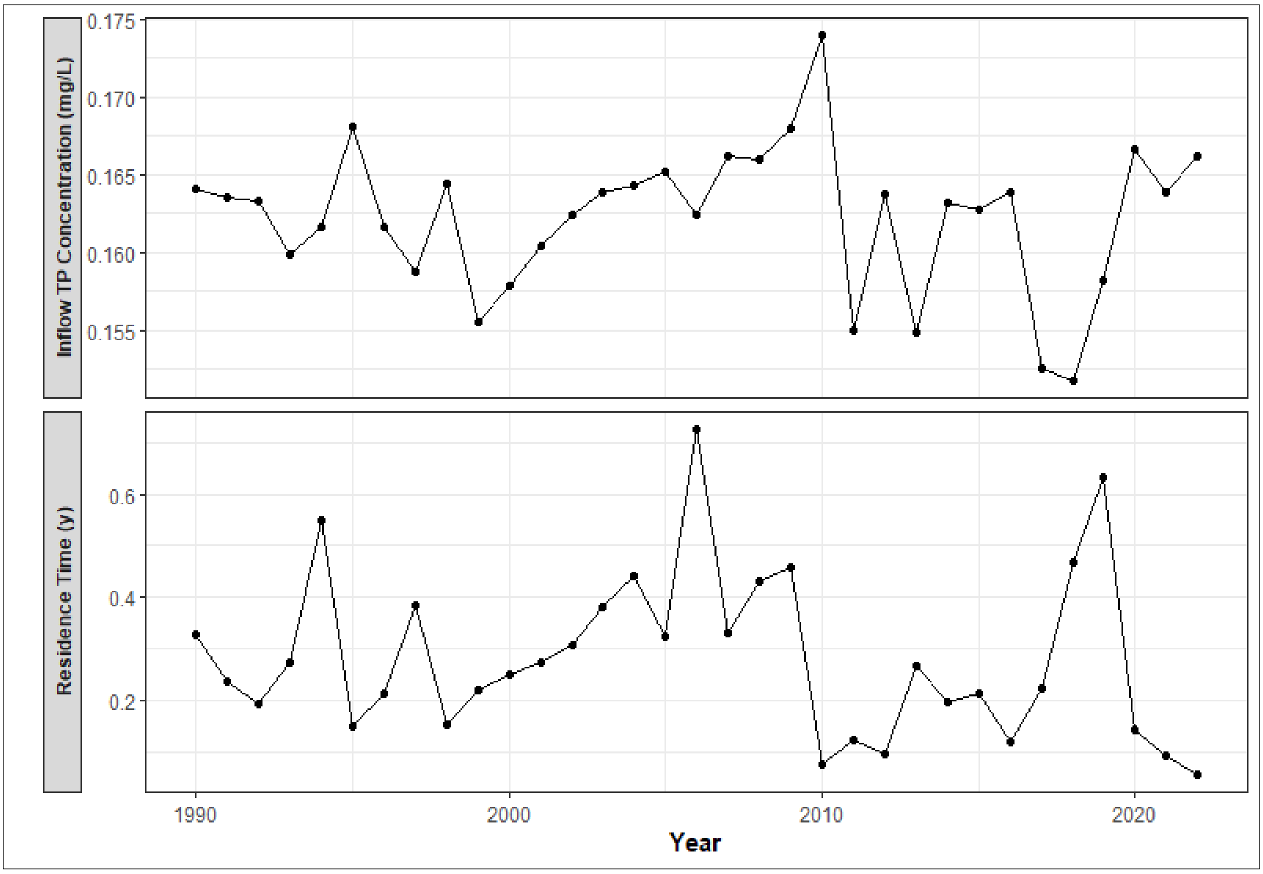

Figure 1. Annual changes in inflow concentrations of total phosphorus (TP_{in}, top panel) and residence time (τ_w, bottom panel) in Lake Tuggeranong, 1990-2022.

2.2 Estimation of total phosphorus and chlorophyll a concentration

Using the modified Vollenweider model (equation 2) TP_{lake} concentration can be predicted using TP_{in} and τ_w values. Before estimating the TP_{lake}, fitting of the constants k and x is required such that modelled TP_{lake} values provide as close-as-possible match to observed TP_{lake} values. Here, our source of observed TP_{lake} values come from the ALS Lakes and Rivers Water Quality Monitoring Program from 2005-2022. This period was chosen because it represents the longest run of annual sampling of LT TP concentrations with relatively constant annual sampling effort (most years had 8 samples from each of two locations within Lake Tuggeranong: LTG248 and LTG249). Other sources of data (e.g. University of Canberra research) did not provide a similar temporal scope, meaning their inclusion would have led to large variability in the underlying sampling effort behind annual averages, which was undesirable here. Waterwatch data was also not used here due to questions about data reliability specifically related to TP measures relative to lab techniques (Harrison *et al.* 2013).

To fit optimal values of k and x to our modified Vollenweider model, an approach similar to Brett & Benjamin (2008) was adopted. Specifically, a function was created in R that would, for given values of k and x, predict lake TP concentrations ($\mathrm{TP_{predicted}}$) and compare them to average annual observed lake TP concentrations $\mathrm{TP_{observed}}$ values to calculate the weighted error sum of squares (WESS) as follows:

$$\mathrm{WESS} = \sum \mathrm{weight} \times \left(\mathrm{TP_{observed}} - \mathrm{TP_{predicted}}\right)^2 \qquad (7)$$

The number of samples that each annual average was based on as weights was used, such that years that had annual averages based on relatively few datapoints (e.g. 2020 due to lockdowns) contributed relatively less to the WESS. This WESS provides a measure of how closely matched predicted and observed TP values are, with low WESS values indicating a better fitting model.

This function was then used alongside a solver function in R to find a combination of k and x values that minimised the WESS and thus represented the best fit to the observed data. This solver yielded values of $k = 1.093$ and $x = -0.791$. These values are similar to those reported elsewhere (e.g. Brett & Benjamin 2008) and not particularly inconsistent with the unmodified Vollenweider model used by Beer *et al.* 1982 in their early work on LT. Sensitivity analysis of this optimisation was performed by re-calculating the

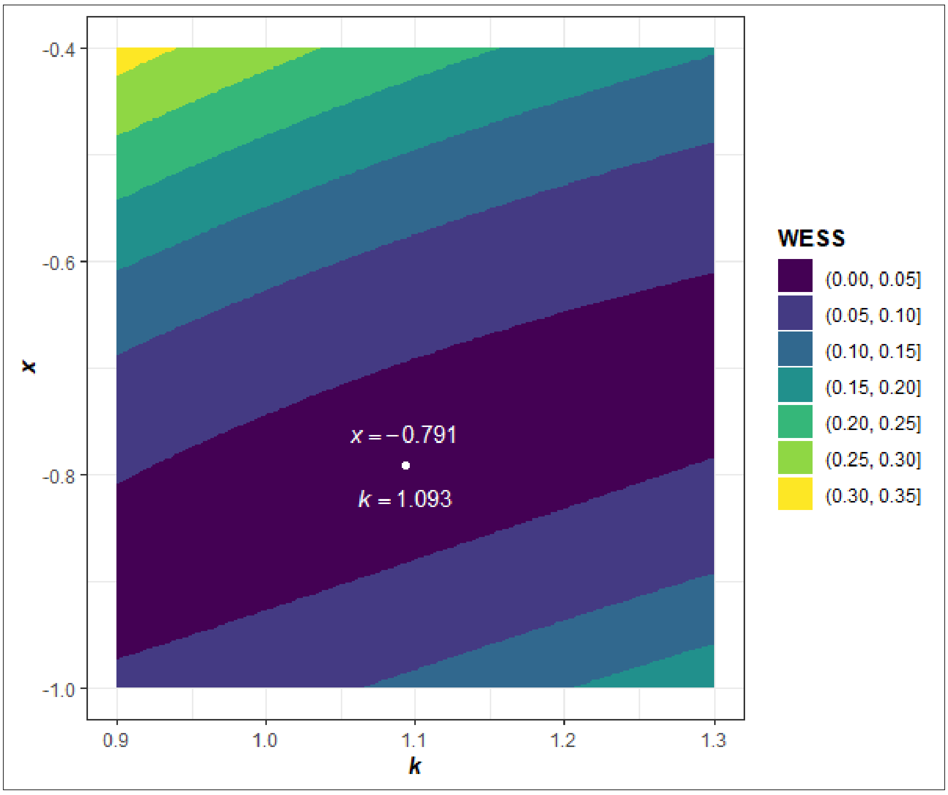

Figure 2. Sensitivity of the Lake Tuggeranong modified Vollenweider model to changes in *k* and *x* in terms of the resultant weighted error sum of squares (WESS). The white point indicates the optimal values.

resultant WESS with a range of k and x values. This analysis indicated that a relatively wide band of k and x values yielded similarly good fits (indicated by relatively low WESS values: Figure 2), which again was consistent with the findings of Brett & Benjamin 2008.

Using the fitted values of k and x, the modified Vollenweider model is represented in equation 8:

$$\mathrm{TP_{lake}} = \frac{\mathrm{TP_{in}}}{1+\left(1.093\times\tau_w^{-0.791}\right)*\tau_w} \qquad (8)$$

This yielded predicted $\mathrm{TP_{lake}}$ values similar to observed values (Table 2, Figure 3).

Table 2. Comparison of observed annual average and predicted total phosphorus concentrations in Lake Tuggeranong ($\mathrm{TP_{lake}}$) from 2005-2022.

	Observed $\mathrm{TP_{lake}}$ (mg/L)	Predicted $\mathrm{TP_{lake}}$ (mg/L)
Minimum	0.064	0.079
Median	0.086	0.090
Maximum	0.114	0.107

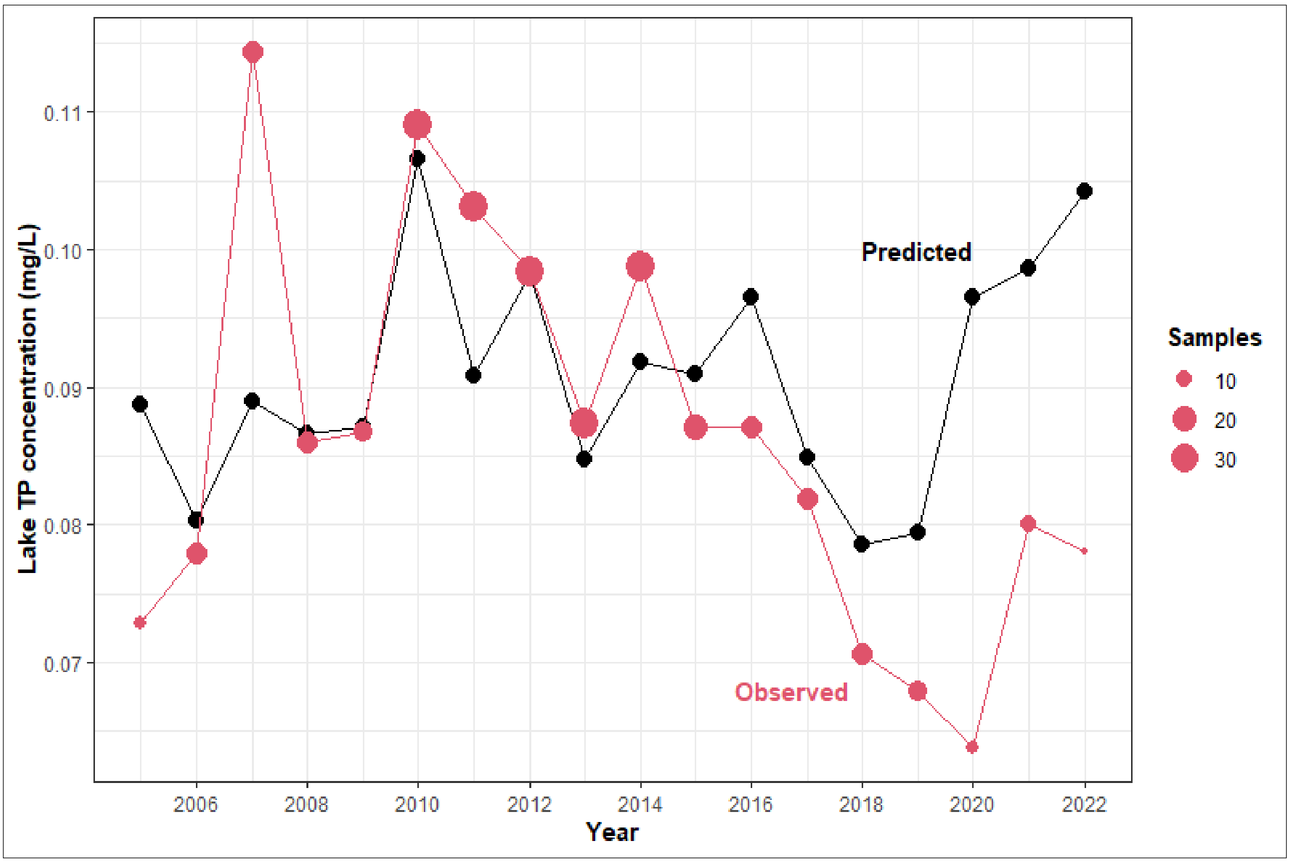

Figure 3. Comparison of observed (red) and predicted (black) annual total phosphorus (TP) concentrations in Lake Tuggeranong from 2005-2022 shows the modified Vollenweider model captures the centre and spread of observed values well with some mirroring of year-to-year trends. Observed values are averages of samples for each year, and data point size represents the number of underlying samples.

The predicted TP concentrations in Lake Tuggeranong had a very similar average and variability to observed TP concentrations, and for many years the two time-series followed similar up/down-wards trajectories (Figure 3). Overall, predicted and observed TP concentrations in Lake Tuggeranong were positively correlated. This was confirmed by a linear regression model relating the two (weighted based on annual sample number), which yielded a significant association between the two variables (ANOVA F-test of slope coefficient: F=8.5889, p=0.0098), and an adjusted r^2 value of 0.3086.

This model can be used to predict the chlorophyll a level using the power relationship described by Lawrence (1980) and applied in Beer *et al.* (1982) in their early work on Lake Tuggeranong:

$$\text{Chlorophyll a} = 0.367 * TP_{Lake}^{0.91} \qquad (9)$$

This relationship was derived from a comparison of several Canberra lakes.

3. Estimation of Lake Trophic State

Using 133 years of annually aggregated Source model output of TP load and inflow water volumes, the above-mentioned models were combined under a range of influent TP reduction scenarios ranging from current/business as usual (BAU: no reduction in TP load), and 100kg to 900kg reduction in annual TP load coming into the lake (with no change to inflow water volumes). Additionally, a predevelopment flow and TP load scenario was considered, which assumed a wholly undeveloped catchment. The Vollenweider model was used to predict annual TP concentrations in the lake for each year under each scenario. This was then converted to predicted chlorophyll a concentration using equation 9. Finally, these predicted concentrations were compared to published cut-offs (e.g. Neil 2012) of chlorophyll a concentration under different trophic states: Oligotrophic <5 µg/L, Mesotrophic <15 µg/L, Eutrophic >15 µg/L.

The predictions for the predevelopment scenario show that, in the absence of development within the catchment, Lake Tuggeranong would likely have experienced a wide range of trophic conditions. In some years the lake would not have received any inflow at all and would likely have reached an oligotrophic state. But in high rainfall years the lake's size and the inflow of TP would have pushed it towards a eutrophic state. This provides a good baseline for considering realistic futures for the lake.

Under the 'current' scenario (BAU), Lake Tuggeranong is predicted to (at the annual, average timescale) remain in a eutrophic state (Figure 4). To avoid a constant eutrophic state, the results of the TP reduction scenarios show that TP load reductions of 600kg or more into Lake Tuggeranong are predicted to result in conditions that are eutrophic less often than not (Figure 4). However, even under high TP load reduction scenarios there will be years of high TP inflow (wet years) where we probably cannot avoid eutrophic conditions developing (as with the pre-development scenario) simply because of the large total TP load entering the lake via stormwater (such that even a >600kg reduction in TP is proportionally fairly insignificant). Conversely, though, dry years with low TP inflows could yield oligotrophic conditions in scenarios with greater than 500kg TP reduction (Figure 4).

Overall, these modelling results suggest that an annual 700kg reduction in TP entering Lake Tuggeranong is an appropriate target for future lake management plans. This level of TP reduction is predicted to lead to favourable conditions for human recreation in the lake in ~75% of years (~25% oligotrophic, ~50% mesotrophic; Figure 4).

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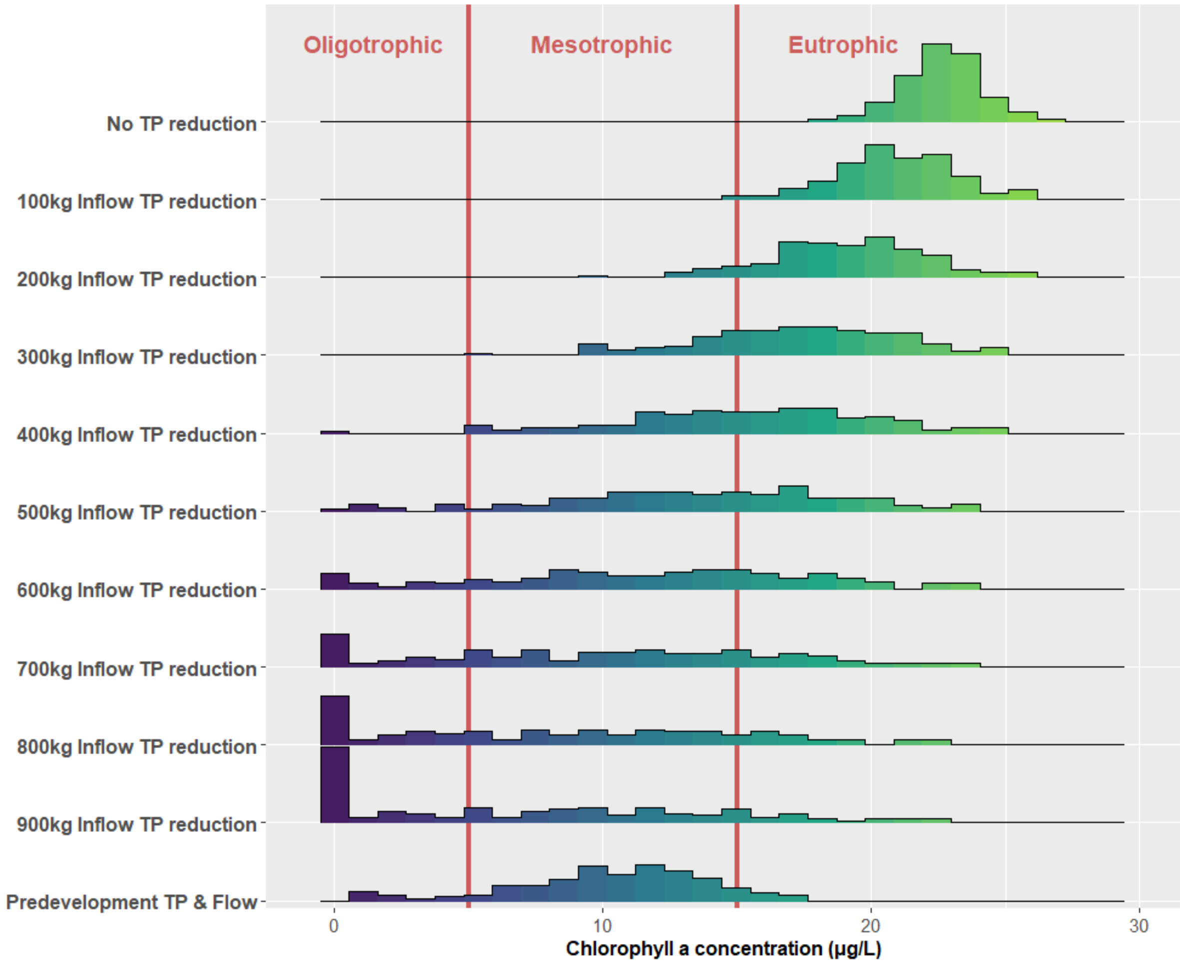

Figure 4. Distribution of annual predicted chlorophyll a concentration (i.e. trophic state) in Lake Tuggeranong under different scenarios of reductions in inflow total phosphorus (TP).

4. Conclusion

Vollenweider's mass-balance model is a simple, but useful, tool for predicting nutrient concentrations in lakes based on influent volumes and concentrations. Whilst they are typically used for comparisons between lakes, in this study the Vollenweider model was modified to derive Lake Tuggeranong specific total phosphorous concentrations. The model produced a reasonable fit with observed values over a 17-year period and could be used in conjunction with catchment model inputs to predict general trends in phosphorous levels in Lake Tuggeranong into the future. The model output was then used to provide an estimate chlorophyll a which is a measure of the trophic state. The estimated chlorophyll a concentration was used to determine the trophic state of the lake under different scenarios. The findings provide an approximate estimate of the target inflow TP load for Lake Tuggeranong to improve the lake trophic conditions, with a target annual reduction of ~700kg of TP yielding favourable predictions.

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